

Repulsive Casimir Force: Sufficient Conditions

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Abstract

In this paper the Casimir energy of two parallel plates made by materials of different penetration depth and no medium in between is derived. We study the Casimir force density and derive analytical constraints on the two penetration depths which are *sufficient conditions* to ensure repulsion. Compared to other methods our approach needs no specific model for dielectric or magnetic material properties and constitutes a complementary analysis.

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I. INTRODUCTION

One of the striking features of the Casimir effect [1, 2] is the dependence of the sign of the energy, which may be positive or negative, and of the corresponding repulsive or attractive force, on the geometry of the device and on the materials used. The possibility of obtaining a repulsive force would open vast possibilities in the design of MEMS and NEMS [3, 4]. For example one of the principal causes of malfunctioning in MEMS is stiction: the collapse of nearby surfaces, resulting in their permanent adhesion. The possibility of having a repulsive Casimir force is an interesting way to avoid such a collapse of the structure but up to now there are only few experimental evidences for a repulsive force [5–7]

In this paper we study the Casimir energy of two parallel plates made by materials having different penetration depth. A study of the Casimir force and the role of surface plasmons between dissimilar mirrors has been carried out in the past for specific models of the dielectric and magnetic properties of the materials [8]. Here we propose an alternative method which is model independent and gives thus complementary information on the possibility of repulsive Casimir forces. We find that, depending on the relation between the penetration depths and the distance of the plates the force can be both attractive and repulsive. The penetration depth of materials can be taken into account by means of its connection to the surface impedance [9–11]. The surface impedance Z of any planar surface may be defined as the ratio of the complex electric and magnetic tangential field components at the surface [9]:

$$\mathbf{E}_t(z_0) = Z(\mathbf{H}_t(z_0) \times \hat{n}) \quad (1)$$

where \hat{n} is a normal vector pointing inside the surface and z_0 is the position of the surface. The main advantage of this formula is that it relates the tangential fields *outside the material*, thus it is not necessary to consider the internal degrees of freedom of the material which are taken into account through the values of Z [9]. Equation (1) can be seen as an exact functional definition of the surface impedance so that it can be applied to arbitrary materials [12] and it still holds when a description in terms of dielectric permittivity cannot be given [13]. Indeed a complete correspondence with reflection coefficient and surface impedance exists [9, 12]. Moreover [3] “*for large permeability and permittivity, the transition from attractive to repulsive behavior depends only on the impedance Z* ”

The paper is organized as follows: in section II the Casimir force in the general configuration is evaluated and some limiting results are recovered. In section III the conditions for

having repulsions are derived. Finally section IV contains remarks and conclusions.

II. THE CASIMIR FORCE

In the following we will consider two parallel plates lying in the (x, y) plane, located at $z = 0, z = a$ and characterized by different surface impedances $Z_{(0,a)}(\omega)$ respectively. Given the functions $\delta_{(0,a)}(\omega) = i \frac{Z_{(0,a)}(\omega)}{\omega}$, $Re[\delta(\omega)]$ is interpreted as the penetration depth of the material at the frequency ω [10, 11] see also [13]. Because of translational invariance in the (x, y) plane the electric and magnetic fields can be written as (in the following we will use natural units: $\hbar = c = 1$):

$$\mathbf{E}(\mathbf{x}, t) = \mathbf{f}(z)e^{ik_{\perp} \cdot x_{\perp} - \omega t}, \quad \mathbf{B}(\mathbf{x}, t) = \mathbf{g}(z)e^{ik_{\perp} \cdot x_{\perp} - \omega t} \quad (2)$$

with $k_{\perp} \equiv (k_x, k_y)$ and $x_{\perp} \equiv (x, y)$. The Maxwell equations imply:

$$\frac{d^2 \mathbf{f}}{dz^2} + \lambda^2 \mathbf{f} = 0; \quad \frac{d^2 \mathbf{g}}{dz^2} + \lambda^2 \mathbf{g} = 0; \quad (3)$$

with $\lambda^2 = \omega^2 - k_x^2 - k_y^2$. Imposing relation (1) we obtain, for the (x, y) components of \mathbf{E} and \mathbf{B} the following boundary conditions [11, 13].

$$\begin{cases} f_x(0) = -\delta_0(ik_x f_z(0) - f'_x(0)) \\ f_x(a) = \delta_a(ik_x f_z(a) - f'_x(a)) \end{cases} \quad \begin{cases} f_y(0) = -\delta_0(ik_y f_z(0) - f'_y(0)) \\ f_y(a) = \delta_a(ik_y f_z(a) - f'_y(a)) \end{cases} \quad (4)$$

moreover everywhere $\nabla \cdot \mathbf{E} = \mathbf{0}$ must be satisfied.

In this way, with a suitable choice of the reference frame, we find the following dispersion equation:

$$\Delta_{TM} \equiv [\delta_0 \delta_a (\lambda^4 + k_{\perp}^4) + \lambda^2 (2\delta_0 \delta_a k_{\perp}^2 - 1)] \sin(a\lambda) - (\delta_0 + \delta_a) \lambda (\lambda^2 + k_{\perp}^2) \cos(a\lambda) = 0 \quad (5)$$

for the TM modes and

$$\Delta_{TE} \equiv \delta_0 \delta_a (\lambda^2 - 1) \sin(a\lambda) - (\delta_0 + \delta_a) \lambda \cos(a\lambda) = 0 \quad (6)$$

for the TE ones. We use the argument theorem to obtain the Casimir energy [2, 14] so that, after ω -rotation to the imaginary axis: $\omega \rightarrow i\zeta$, we have (for the properties of δ (or Z) along the imaginary axis see [10, 11, 15])

$$E = \frac{1}{2(2\pi)^3} \int d\zeta dk_x dk_y \ln [\Delta_{TM}(\lambda, i\zeta) \Delta_{TE}(\lambda, i\zeta)] \quad (7)$$

This integral diverges and, as usual, to regularize it we must subtract the energy corresponding to the configuration with the two plates infinitely far away ($a \rightarrow \infty$):

$$\Delta_{TE}^\infty = -ie^{aq} \left(\frac{1}{2} \delta_0 \delta_a q^2 - \frac{\delta_0 q}{2} - \frac{\delta_a q}{2} + \frac{1}{2} \right) \quad (8)$$

$$\Delta_{TM}^\infty = -\frac{ie^{aq} (\delta_0 q^2 - q - \delta_0 k_x^2) (\delta_a q^2 - q - \delta_a k_x^2)}{2q^2} \quad (9)$$

with $q = \sqrt{\zeta^2 + k_\perp^2}$. Thus the renormalized Casimir energy will be given by:

$$E_R = \frac{1}{2(2\pi)^3} \int_{-\infty}^{\infty} d\zeta dk_x dk_y \ln \left[\left(1 - e^{-2aq} \frac{(1 - q\delta_a(i\zeta)) (1 - q\delta_0(i\zeta))}{(1 + q\delta_a(i\zeta)) (1 + q\delta_0(i\zeta))} \right) \right] + \ln \left[\left(1 - e^{-2aq} \frac{(q - \delta_a(i\zeta)\zeta^2) (q - \delta_0(i\zeta)\zeta^2)}{(q + \delta_a(i\zeta)\zeta^2) (q + \delta_0(i\zeta)\zeta^2)} \right) \right] \quad (10)$$

or, in dimensionless variables

$$E_R = \frac{1}{4\pi^2 a^3} \int_0^\infty d\xi \int_0^\infty k_\perp d\tilde{k}_\perp \ln \left[\left(1 - e^{-2p} \frac{(1 - p\tilde{\delta}_a) (1 - p\tilde{\delta}_0)}{(1 + p\tilde{\delta}_a) (1 + p\tilde{\delta}_0)} \right) \right] + \ln \left[\left(1 - e^{-2p} \frac{(p - \tilde{\delta}_a \xi^2) (p - \tilde{\delta}_0 \xi^2)}{(p + \tilde{\delta}_a \xi^2) (p + \tilde{\delta}_0 \xi^2)} \right) \right] \quad (11)$$

with $p = aq$, $\tilde{k}_{(x,y)} = ak_{(x,y)}$, $\xi = a\zeta$, $\tilde{\delta}_{(a,0)} = \frac{\delta_{(a,0)}}{a}$.

In the following we will concentrate on the Casimir force, it can be written:

$$F_R = -\frac{1}{2\pi^2 a^4} \int_0^\infty d\xi \int_\xi^\infty p^2 dp \frac{e^{-2p} (\tilde{\delta}_0 p - 1) (\tilde{\delta}_a p - 1)}{(\tilde{\delta}_0 p + 1) (\tilde{\delta}_a p + 1) \left(1 - \frac{e^{-2p} (\tilde{\delta}_0 p - 1) (\tilde{\delta}_a p - 1)}{(\tilde{\delta}_0 p + 1) (\tilde{\delta}_a p + 1)} \right)} + \frac{e^{-2p} (p - \tilde{\delta}_0 \xi^2) (p - \tilde{\delta}_a \xi^2)}{(\tilde{\delta}_0 \xi^2 + p) (\tilde{\delta}_a \xi^2 + p) \left(1 - \frac{e^{-2p} (p - \tilde{\delta}_0 \xi^2) (p - \tilde{\delta}_a \xi^2)}{(\tilde{\delta}_0 \xi^2 + p) (\tilde{\delta}_a \xi^2 + p)} \right)} \quad (12)$$

Now it is not difficult to show that the contribution coming from the point $\xi = 0$ is zero, thus we can safely remove this point from the integral, which allows us to rewrite the integral:

$$F_R = -\frac{1}{2\pi^2 a^4} \int_0^\infty d\xi \int_\xi^\infty p^2 dp \sum_{n=1}^{\infty} e^{-2pn} \left[\frac{(\tilde{\delta}_0 p - 1) (\tilde{\delta}_a p - 1)}{(\tilde{\delta}_0 p + 1) (\tilde{\delta}_a p + 1)} \right]^n + e^{-2pn} \left[\frac{(p - \tilde{\delta}_0 \xi^2) (p - \tilde{\delta}_a \xi^2)}{(\tilde{\delta}_0 \xi^2 + p) (\tilde{\delta}_a \xi^2 + p)} \right]^n =: \sum_{n=1}^{\infty} F_R^n \quad (13)$$

The absolute values of the terms in the brackets are always less or equal to one, in the case $\tilde{\delta}_0 = \tilde{\delta}_a = (0, \infty)$ they are maxima (1) and we have:

$$\begin{aligned} F_R &= - \sum_{n=1}^{\infty} \frac{1}{\pi^2 a^4} \int_0^{\infty} d\xi \int_{\xi}^{\infty} p^2 dp e^{-2pn} = - \sum_{n=1}^{\infty} \frac{1}{\pi^2 a^4} \int_0^{\infty} d\xi \frac{e^{-2n\xi}(2n\xi(n\xi + 1) + 1)}{4n^3} \\ &= - \sum_{n=1}^{\infty} \frac{3}{8a^4 n^4 \pi^2} = - \frac{\pi^2}{240a^4}. \end{aligned} \quad (14)$$

In contrast, if we take $\tilde{\delta}_0 = \infty$, $\tilde{\delta}_a = 0$ or viceversa they take on minimum values of (-1) and we obtain

$$\begin{aligned} F_R &= - \sum_{n=1}^{\infty} \frac{1}{\pi^2 a^4} \int_0^{\infty} d\xi \int_{\xi}^{\infty} p^2 dp (-1)^n e^{-2pn} = \\ &= - \sum_{n=1}^{\infty} \frac{3(-1)^n}{8a^4 n^4 \pi^2} = \frac{7}{8} \frac{\pi^2}{240a^4}. \end{aligned} \quad (15)$$

Thus, in this case we recover the result obtained by Boyer [16] for two non dispersive mirrors having $\epsilon = (\infty, 1)$, $\mu = (1, \infty)$ respectively, see also [8, 17]. The upper calculation also constitutes an independent demonstration of the result found by Henkel and Joulain eq.(4) of [17].

From eq. (13) we may understand intuitively what kind of conditions must be satisfied to have repulsion. Indeed, if the two slabs are made of the same material we have $\tilde{\delta}_a = \tilde{\delta}_0$ and the expression of the force becomes:

$$F_R = - \sum_{n=1}^{\infty} \frac{1}{2\pi^2 a^4} \int_0^{\infty} d\xi \int_{\xi}^{\infty} p^2 dp e^{-2pn} \left[\left(\frac{\tilde{\delta}_0 p - 1}{\tilde{\delta}_0 p + 1} \right)^{2n} + \left(\frac{p - \tilde{\delta}_0 \xi^2}{\tilde{\delta}_0 \xi^2 + p} \right)^{2n} \right]. \quad (16)$$

In this case the integrand is always positive and the force will be always attractive. The only possibility to have repulsion is to have $\tilde{\delta}_a \neq \tilde{\delta}_0$, such that $\left[\frac{(\tilde{\delta}_0 p - 1)(\tilde{\delta}_a p - 1)}{(\tilde{\delta}_0 p + 1)(\tilde{\delta}_a p + 1)} \right]^n + \left[\frac{(p - \tilde{\delta}_0 \xi^2)(p - \tilde{\delta}_a \xi^2)}{(p + \tilde{\delta}_0 \xi^2)(p + \tilde{\delta}_a \xi^2)} \right]^n$ be negative. Fortunately, the series starts with the term $n = 1$ so that the possibility is not ruled out.

In the next section we will study the case $\tilde{\delta}_a \ll 1$ and we will concentrate on the first term of the series: $n = 1$.

III. SUFFICIENT CONDITIONS FOR REPULSION

In the following we will develop F_R^1 at first order around $\tilde{\delta}_a = 0$. After the integration on the p variable we will study the behavior of the remaining integrand which will be a function

of ξ only and determine what conditions must be satisfied to have repulsion.

$$\begin{aligned}
F_R^1 &= \frac{1}{2\pi^2 a^4} \int_0^\infty d\xi \int_\xi^\infty p^2 dp e^{-2p} \left[(2p\tilde{\delta}_a - 1) \left(1 - \frac{2\tilde{\delta}_0 p}{\tilde{\delta}_0 p + 1} \right) + \left(2\tilde{\delta}_a \frac{\xi^2}{p} - 1 \right) \left(1 - \frac{2\tilde{\delta}_0 \xi^2}{\tilde{\delta}_0 \xi^2 + p} \right) \right] \\
&= \frac{1}{8\pi^2 a^4} \int_0^\infty d\xi \frac{e^{-2\xi}}{\tilde{\delta}_0^3} \left\{ I_1(\xi) - 8(1 + 2\tilde{\delta}_a) I_2 \left(2\xi + \frac{2}{\tilde{\delta}_0} \right) + 8\tilde{\delta}_0^5 (\tilde{\delta}_0 + 2\tilde{\delta}_a) \xi^6 I_2 \left(2\xi + 2\tilde{\delta}_0 \xi^2 \right) \right\} \\
&=: \frac{1}{128\pi^2 a^4} \int_0^\infty d\xi f_R^1(\xi)
\end{aligned} \tag{17}$$

With:

$$\begin{aligned}
I_1(\xi) &= -4\xi^4 \tilde{\delta}_0^5 + \left(4\xi^3 + 2\xi^2 - 2\tilde{\delta}_a \left(2\xi(\xi + 1) (2\xi^2 + 3) + 3 \right) \right) \tilde{\delta}_0^4 + \\
&\quad \tilde{\delta}_a (2\xi(4\xi(\xi + 2) + 7) + 7) \tilde{\delta}_0^3 + (\tilde{\delta}_a(-8\xi - 4) - 4\xi - 2) \tilde{\delta}_0^2 + (8\tilde{\delta}_a + 4) \tilde{\delta}_0 \\
I_2(\xi) &= e^\xi E_1(\xi)
\end{aligned}$$

$E_n(x)$ is the *exponential integral function* [19]. Let us study $f_R^1(\xi)$ for the two regimes, $0 \leq \tilde{\delta}_a < \tilde{\delta}_0 \ll 1$ and $0 \leq \tilde{\delta}_a \ll 1, \tilde{\delta}_0 \gg 1$. In the first case we find:

$$F_R^1 = \frac{1}{8\pi^2 a^4} \int d\xi e^{-2\xi} \left[(\tilde{\delta}_0 + \tilde{\delta}_a) (8\xi^3 + 8\xi^2 + 6\xi + 3) - (2\xi^2 + 2\xi + 1) \right] \tag{18}$$

Thus, in the range of frequencies relevant to the Casimir effect, $\xi \sim 1$, the condition for having repulsion is, at second order in ξ :

$$\tilde{\delta}_0 > \frac{2 + 4\xi(1 + \xi)}{3 + 2\xi(3 + 4\xi(1 + \xi))} - \tilde{\delta}_a \sim -\tilde{\delta}_a + 0.775 - 0.494\xi + 0.119\xi^2. \tag{19}$$

This shows that it would be possible to have repulsion if $\tilde{\delta}_a \approx 0$ and $\tilde{\delta}_0 > 0.4$. However this last condition is in contradiction with the assumption $\tilde{\delta}_0 \ll 1$. Let us also note that if we assume $\tilde{\delta}_a = \tilde{\delta}_0 = \text{const}$ we can evaluate all terms of the series (13) and recover the result of Mostepanenko and Trunov [11], (see also [18] for equivalent results for a scalar field).

If $0 \leq \tilde{\delta}_a \ll 1, \tilde{\delta}_0 \gg 1$ we find at first order (for the asymptotic expansion of $E_n(\xi)$ see [19])

$$F_R^1 = \frac{1}{16\pi^2 a^4} \int d\xi \frac{e^{-\xi}}{\tilde{\delta}_0 \xi^2} \left\{ -\tilde{\delta}_0^2 A(\tilde{\delta}_a, \xi) + \tilde{\delta}_0 B(\tilde{\delta}_a, \xi) + C(\tilde{\delta}_a, \xi) \right\} \tag{20}$$

with:

$$\begin{aligned}
A(\tilde{\delta}_a, \xi) &= -2\tilde{\delta}_a \xi^2 (3 + 2\xi(3 + \xi(3 + 2\xi))) \\
B(\tilde{\delta}_a, \xi) &= 2\xi^2 + 4\xi^3(1 + \xi) + \tilde{\delta}_a \xi^2 (7 + 2\xi(7 + 6\xi)) \\
C(\tilde{\delta}_a, \xi) &= -3 - 6\xi - 8\xi^2 - 8\xi^3 + 8\tilde{\delta}_a \xi^4
\end{aligned}$$

The term within the curly brackets in Eq. (20) is a second order polynomial in $\tilde{\delta}_0$ and, to have repulsion, it must be positive. Since the coefficient of the $\tilde{\delta}_0^2$ term is always negative we must require that the discriminant of the associated second order equation is positive. This discriminant is a second order polynomial in $\tilde{\delta}_a$ and we have to study the associated second order equation:

$$\tilde{\delta}_a^2 D(\xi) + \tilde{\delta}_a E(\xi) + F(\xi) = 0 \quad (21)$$

with

$$\begin{aligned} D(\xi) &= 49\xi^4 + 196\xi^5 + 556\xi^6 + 720\xi^7 + 528\xi^8 + 256\xi^9 \\ E(\xi) &= -72\xi^2 - 288\xi^3 - 596\xi^4 - 848\xi^5 - 744\xi^6 - 432\xi^7 - 160\xi^8 \\ F(\xi) &= 4\xi^4 + 16\xi^5 + 32\xi^6 + 32\xi^7 + 16\xi^8. \end{aligned}$$

Since $D(\xi) > 0$, for the polynomial to be positive we have to choose $\tilde{\delta}_a$ such that it lies outside the interval defined by the two roots $\tilde{\delta}_a^{min}$ and $\tilde{\delta}_a^{max}$ of the corresponding equation, that is $\tilde{\delta}_a < \tilde{\delta}_a^{min}$ or $\tilde{\delta}_a > \tilde{\delta}_a^{max}$ for $G = E^2 - 4DF > 0$. If $G \leq 0$ we may take any value for $\tilde{\delta}_a$. In that case the two roots coincide if $\tilde{\delta}_a = 0$ and become imaginary of $\tilde{\delta}_a > 0$ excluding any physical solution. Fig.1 illustrates the behavior of the two roots as a function of imaginary frequency.

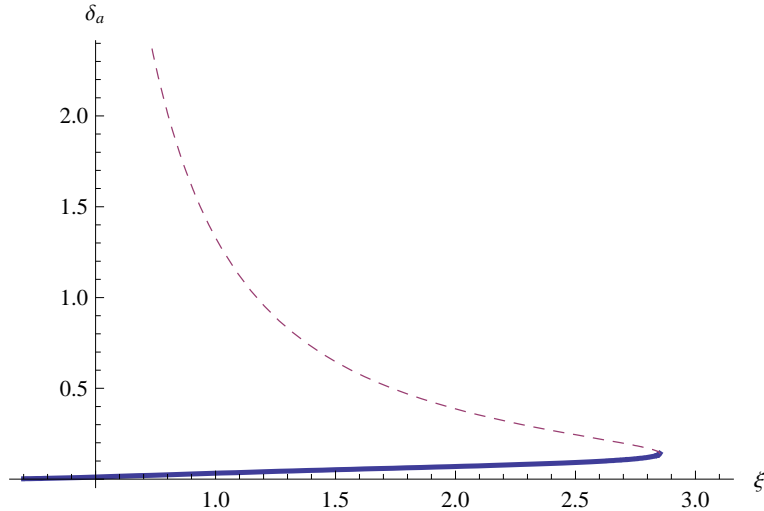


FIG. 1. The two roots of eq. (21), $\tilde{\delta}_a^{min}$ and $\tilde{\delta}_a^{max}$, are shown as function of imaginary frequency ξ (solid and dashed curve respectively). For $\xi \geq 2.85$, $G \leq 0$, at $\xi = 2.85$ we have $\tilde{\delta}_a^{min} = \tilde{\delta}_a^{max}$ and at larger frequency any value of $\tilde{\delta}_a$ will give rise to a positive value for (21).

Since $\tilde{\delta}_a^{max}$ can be larger than 1 (see Fig.1) and we assumed $\tilde{\delta}_a \ll 1$ we remain with the only possibility $\tilde{\delta}_a < \tilde{\delta}_a^{min}$. Around $\xi \sim 1$ the condition for the positivity can be written:

$$\tilde{\delta}_a < \tilde{\delta}_a^{min} = \frac{-E - 2\sqrt{E^2 - 4DC}}{2D} \sim -0.0135 + 0.05148\xi - 0.00533\xi^2. \quad (22)$$

If this inequality is satisfied the force density will be repulsive for those values of $\tilde{\delta}_0$ which satisfy

$$\tilde{\delta}_0^{min} \leq \tilde{\delta}_0 \leq \tilde{\delta}_0^{max}, \quad (23)$$

$\tilde{\delta}_0^{min}$ ($\tilde{\delta}_0^{max}$) being the smaller (larger) roots of the associated equation:

$$\begin{aligned} -\tilde{\delta}_0^2 A(\tilde{\delta}_a, \xi) + \tilde{\delta}_0 B(\tilde{\delta}_a, \xi) + C(\tilde{\delta}_a, \xi) &= 0 \\ \tilde{\delta}_0^{min} &= \frac{-B + \sqrt{B^2 - 4AC}}{2A} \sim a_1 + \tilde{\delta}_a a_2 \\ \tilde{\delta}_0^{max} &= \frac{-B - \sqrt{B^2 - 4AC}}{2A} \sim a_3 - \frac{1}{\tilde{\delta}_a} a_4 - \tilde{\delta}_a a_2 \end{aligned} \quad (24)$$

where

$$\begin{aligned} a_1 &= 10.48 - 12.56\xi + 4.58\xi^2, \quad a_2 = 152.438 - 230.856\xi + 93.118\xi^2, \\ a_3 &= -9.16122 + 12.0305\xi - 4.50083\xi^2, \quad a_4 = 0.376294 - 0.126549\xi + 0.013413\xi^2 \end{aligned}$$

Note that in the case of an ideal mirror at $z = a$ we have $\tilde{\delta}_a = 0$, $\tilde{\delta}_0^{max} \rightarrow \infty$ and the only condition to be satisfied is:

$$\tilde{\delta}_0 > \tilde{\delta}_0^{min} = 10.48 - 12.56\xi + 4.58\xi^2.$$

Thus the situation in which one mirror is ideal gives rise to quite different results than the ones obtained when both are real. When both mirrors are real, they both must satisfy restrictions to ensure repulsion and, moreover, a precise relation between the two penetration depths must be fulfilled (eqs. (22,23)).

IV. DISCUSSION AND CONCLUSIONS

Let us apply our results to one known situation, that is of two mirrors described by the plasma model. In this case we have

$$\tilde{\delta}_{(0,a)} = \frac{1}{\sqrt{(\xi_{(0,a)}^p)^2 + \xi^2}} \quad \text{with} \quad \xi_{(0,a)}^p = a\omega_{(0,a)}^p$$

$\omega_{(0,a)}^p$ being the plasma frequency of the mirror in $z = (0, a)$ respectively. Condition (22) gives $\xi_a^p > \frac{1}{0.032} \sim 30.65$ which means that the mirror in a must have a plasma frequency $\omega_a^p \geq 30.65/a$, but condition (23) implies $\frac{1}{\sqrt{(\xi_0^p)^2 + 1}} > 4.37$ which, being $\sqrt{(\xi_0^p)^2 + 1} \geq 1$, is impossible.

Let us consider now the case of hypothetical materials having $\tilde{\delta}_0(\xi) = k/\xi$ with $k = 1, 2, 3, 4$. There we obtain for the Casimir force, using the exact first three terms of the series eq.(13):

$$a^4 F_R(k=1) = -0.0007 - 0.0004 - 0.0000 = -0.0011$$

$$a^4 F_R(k=2) = 0.0084 - 0.0005 + 0.0000 = 0.0079$$

$$a^4 F_R(k=3) = 0.0133 - 0.0006 + 0.0001 = 0.0128$$

$$a^4 F_R(k=4) = 0.0164 - 0.0008 + 0.0001 = 0.0157$$

The result is illustrated on Fig. 2. The left hand part shows $\tilde{\delta}_a^{min}$ (dashed line) and $\tilde{\delta}_a = \frac{1}{\sqrt{30.65^2 + \xi^2}}$ (solid line) as a function of imaginary frequency. In the right hand part the shaded area gives the range of values of $\tilde{\delta}_0$ given by condition (23) for which the Casimir force becomes repulsive while the dashed, dotted, dotted-dashed and solid lines, decreasing monotonously with increasing frequency, represent $\tilde{\delta}_0$ for $k = 1, 2, 3, 4$ respectively. For $k = 2, 3, 4$ the force turns out to be the more repulsive the higher the k -value, even though the values of $\tilde{\delta}_0$ are only on the limit of the favorable region.

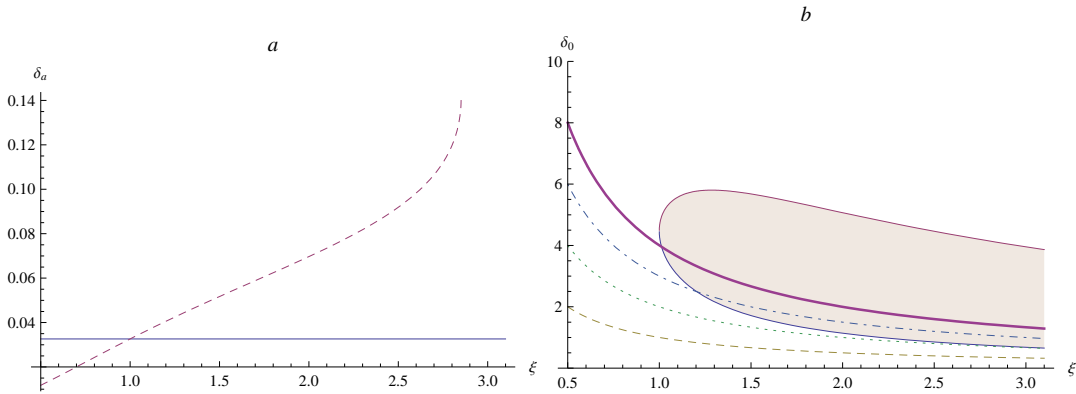


FIG. 2. In a $\tilde{\delta}_a^{min}$ and $\tilde{\delta}_a = \frac{1}{\sqrt{30.65^2 + \xi^2}}$ are shown as dashed and solid curves respectively. In b the area between the two curves $\tilde{\delta}_0^{max}, \tilde{\delta}_0^{min}$ (shaded area) and the curves k/ξ for $k = 1, 2, 3, 4$ dashed, dotted, dash-dotted, and thick respectively are shown. For $\xi < 1$ $\tilde{\delta}_a > \tilde{\delta}_a^{min}$ and consequently the results are imaginary and no physical solution exists.

In conclusion we have found sufficient conditions for the Casimir force to be repulsive with an approach considering only the skin depth and needing no specific model of dielectric or magnetic properties. It would be interesting to study now how much these conditions can be softened, as after all to have a positive integral it is not necessary to have a positive integrand. From this point of view our analysis must be deepened trying to obtain analytical necessary conditions for having a repulsive force. Nonetheless our result demonstrate that repulsion is possible if the penetration depth of the two mirrors satisfy appropriate relations.

The approach seems promising as it can be extended to anisotropic material characterized by a tensorial surface impedance and to more general material [12].

It would also be very interesting to derive the skin depth at optical frequencies from the available tabulated data to search for materials matching the conditions we have established and to use our result to design new materials such as to have repulsive properties.

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